Phys 221 Equations Page

Quadratic formula: if $Az^2 + Bz + C = 0$, then $z = \frac{1}{2A}\left[-B \pm \sqrt{B^2 - 4AC}\right]$

constant acceleration: $\vec{x}(t) = \vec{x}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$, $\vec{v}(t) = \vec{v}_0 + \vec{a}t$

$\vec{x}(t) = \vec{x}_0 + \frac{1}{2}(\vec{v}_0 + \vec{v})t$ $\frac{1}{2}v_f^2 - \frac{1}{2}v_i^2 = \vec{a} \cdot \left(\vec{x}_f - \vec{\vec{x}}_i\right)$

circular motion: $\boldsymbol{a_N = \frac{v^2}{r}}$ (pointing inward) $a_T = rate\ of\ change\ of\ speed$

center of mass: $\boldsymbol{x_{cm} = \frac{\sum m_i x_i}{\sum m_i}}$, $\boldsymbol{y_{cm} = \frac{\sum m_i y_i}{\sum m_i}}$, etc. $\vec{v}_{CM} = \frac{\sum m_i \vec{v}_i}{\sum m_i}$

momentum: $\vec{p} = m\vec{v}$ (and $\vec{p}$ is conserved in the absence of external forces)

$Impulse = \int \vec{F}\, dt$ $Net\ impulse = \int \overrightarrow{\Sigma F}\, dt = \Delta\vec{p}$

force: **2nd law:** $\sum \vec{F} = \frac{d\vec{p}}{dt}$ *or* $\sum \vec{F} = m\vec{a}$ (if mass is constant)

3rd law: $\vec{F}_{AB} = -\vec{F}_{BA}$

work: work $= \vec{F} \cdot \Delta\vec{x} = F\ \Delta x \cos\theta$ (if force is constant) **or** work $= \int \vec{F} \cdot d\vec{x}$

kinetic energy: $K = \frac{1}{2}mv^2$

Net Work and Kinetic Energy: $\sum \vec{F}\ \cdot\ \Delta\vec{x} = \Delta K$ (if force is constant)

or : Work done by the net force $= \int \left(\sum \vec{F}\right) \cdot d\vec{x} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \Delta K$

Gravity and potential energy: $\vec{F} = m\vec{g}$ (pointing down) and $U_{grav} = mgh + C$

Far from Earth: $F = \frac{GMm}{r^2}$ $U_{grav} = -\frac{GMm}{r} + C$

Springs and elastic potential energy: $\vec{F} = -k\ \vec{x}$ and $U_{elastic} = \frac{1}{2}\,k\,x^2$ (+C)

where *x* is measured from equilibrium.

Conservative forces: $F_{U,x} = -\frac{dU}{dx}$ $U(x) = -\int \vec{F} \cdot \overrightarrow{dx}$

kinetic friction: $f_K = \mu_K\ N$, (opposing the direction of motion)

static friction: $f_S \le \mu_S\ N$, (opposing the direction of other forces)

Work done by kinetic friction: $Work = \vec{f}_K \cdot \Delta\vec{x}$ (usually "lost" as waste heat)

Elastic collisions: (measurements in the center of mass frame are marked with a *)

$\vec{v}^*_{a,i} = \vec{v}_{a,i} - \vec{v}_{CM}$ and $\vec{v}_{a,f} = \vec{v}_{CM} - \vec{v}^*_{a,i}$