Homework #1

GRCC Physics 221

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1. Perform each of the following unit conversions and report your answer to the proper number of significant figures. You may use the following information:

0.225 pounds = 1.00 newton	1 mile = exactly 5280 feet
2.54 centimeters = 1.00 inch	1 foot = exactly 12 inches
1 minute = exactly 60 seconds	1 hour = exactly 60 minutes

   a. In October of 2007, in Scotland's Inverness River, not far from the infamous Loch Ness, a 56 inch long salmon was caught. How long is that in centimeters?

$$56\ inches = (56\ inches)\left(\frac{2.54\ cm}{1\ inch}\right) = 140\ cm\ \ (notice\ the\ sig\ figs!)$$

   b. In September of 2010, Cincinnati Reds pitcher Aroldis Chapman threw a baseball with a speed of 105.1 miles per hour. How fast is that in meters per second?

$$105.1\ mph = \left(\frac{105.1\ mi}{hour}\right)\left(\frac{5280\ ft}{1\ mile}\right)\left(\frac{12\ in}{1\ ft}\right)\left(\frac{2.54\ cm}{1\ in}\right)\left(\frac{1\ m}{100\ cm}\right) = 1.69 \times 10^5\ \frac{m}{hr}$$

$$1.69 \times 10^5\ \frac{meters}{hour} = \left(1.69 \times 10^5\ \frac{meters}{hour}\right)\left(\frac{1\ hour}{60\ min}\right)\left(\frac{1\ min}{60\ s}\right) = 46.98\ m/s$$

   c. The SI unit of pressure is the pascal. One pascal (1 Pa) is exactly one newton per square meter. The record for high atmospheric pressure in Seattle was set in 1921 when the barometer read 15.1 pounds per square inch. How much pressure is that in pascals?

$$15.1\ psi = \left(\frac{15.1\ lbs}{inch^2}\right)\left(\frac{1\ newton}{0.225\ lbs}\right)\left(\frac{1\ inch}{2.54\ cm}\right)^2\left(\frac{100\ cm}{1\ m}\right)^2 = 1.04 \times 10^5\ \frac{N}{m^2} = 104\ kPa$$

2. As Hagrid explained to Harry, that there are 29 knuts in a sickle and 17 sickles in a galleon. J. K. Rowling has said that there are five British pounds to a galleon and at the time she said that there were 1.58 US dollars to a British pound. A boxed set of seven hardcover Harry Potter books retails for $195.00. If Hagrid wants to buy the boxed set, what does he have to pay in galleons, sickles, and knuts? (Assume he pays with the maximum number of galleons and the minimum number of knuts.)

$$\$195.00 = (195\ dollars)\left(\frac{1\ Pound}{\$1.58}\right)\left(\frac{1\ galleon}{5\ Pounds}\right) = 24.684\ galleons$$

$$0.684\ galleons = (0.684\ galleons)\left(\frac{17\ sickles}{1\ galleon}\right) = 11.6\ sickles$$

$$0.6\ sickles = (0.6\ sickles)\left(\frac{29\ knuts}{1\ sickle}\right) = 18\ knuts$$

So the books cost 24 galleons, 11 sickles, and 18 knuts.

[Notice the reduction in significant figures in each line of the calculation!]

3. One fathom is 1.8 meters. One degree Celsius is 1.8 degrees Fahrenheit.
   a. A length of 7.0 fathoms is equal to how many meters?

$$7.0\ fathoms = (7.0\ fathoms)\left(\frac{1.8\ meters}{1\ fathom}\right) = 12.6\ m\ \ (or\ 13\ m, either\ answer\ is\ okay)$$

   b. A temperature of 7.0 degrees Celsius is equal to what temperature Fahrenheit?

$$7.0°\ C = (7.0°\ C)\left(\frac{1.8\ °\text{F}}{1\ °\text{C}}\right) + 32°\text{F} = 44.6\ °\text{F}\ \ (or\ 45°\text{F})$$

   c. Why are the previous two answers different?

Temperature is an unusual quantity since scales employ different units and also a different choice of zero point. If an object is "0 cm" long then it is also "0 miles" long and also "0 ft" long, but a temperature of 0°F is not the same as a temperature of 0°C. This difference stems from a comparatively weak understanding of the concept of "temperature" at the time that temperature scales were invented.

4. The “cubit” is a unit of length that was common in biblical times. The Bible says that Solomon made a circular tank that was ten cubits across and thirty cubits around. This is often reported as an error in the Bible but think about it. Is it actually wrong? Explain your answer.

   Strictly speaking, Solomon was completely correct and not just because they didn’t have a good value for π in his day. The diameter of the circle is known to one significant figure and so the circumference should only be specified to one significant figure as well. The uncertainty in the stated diameter is great enough that a value for the circumference which appears more precise (such as 31.4 cubits) would actually be incorrect.

5. Rainfall is measured in “centimeters” or “inches.”
   a. In 1947 in the town of Holt, Missouri, it rained 12 inches in just 42 minutes. One acre is 43560 square feet. There are 7.48 gallons in a cubic foot. How many gallons of water fell on each acre of land in Holt, Missouri, in those 42 minutes?

      By “an inch of rainfall” we mean enough rain that a watertight container would be filled to a depth of one inch. It does not matter whether the container is a test tube, a soup can, or a swimming pool. If the area at the top of the container is the same as the area at the base then each container would be filled to the same depth. The same would be true of a container with a surface area of one acre. Thus the amount of water per acre was:

$$Volume = area \times depth = (1\ acre)(12\ inches) = (43560\ ft^2)(1\ ft) = 43560\ ft^3$$

$$43560\ ft^3 = (43560\ ft^3)\left(\frac{7.48\ gal}{1\ ft^3}\right) = 3.26 \times 10^5\ gallons$$

   b. In 1956 in the town of Unionville, Maryland, it rained 1.23 inches in just one minute. How many gallons of water fell on each acre of Unionville in that minute?

$$Volume = (1\ acre)(1.23\ in) = (43560\ ft^2)(1.23\ in)\left(\frac{1\ ft}{12}\right) = 4.46 \times 10^3\ ft^3$$

$$4.46 \times 10^3\ ft^3 = (4.46 \times 10^3\ ft^3)\left(\frac{7.48\ gal}{1\ ft^3}\right) = 3.34 \times 10^4\ gallons$$

6. The density of sodium chloride is approximately 2.165 grams per cubic centimeter. The atomic mass of sodium is approximately 23.0. The atomic mass of chlorine is approximately 35.5 To this degree of accuracy, the atomic mass of an ion is the same as the atomic mass of an atom. Sodium chloride forms a "simple cubic" lattice which is to say that the atoms (or ions) lie on the corners of a cubic grid, like the integer-valued points on a Cartesian x-y-z coordinate system. Exactly half of the ions in sodium chlorine are sodium ions and the other half are chlorine ions. The sodium ions take up the same amount of space as the chlorine ions. Since a cube has six square faces, each sodium ion has six chlorine ions for neighbors. Each chlorine ion as six sodium ions for neighbors.
    a. How many sodium ions are there in 1.00 cubic centimeter of sodium chloride?

    A little knowledge of chemistry, or at least of atoms, is needed. If the atomic mass of sodium is 23.0 that means that one "mole" of sodium atoms would have a mass of 23.0 grams (yes, this is no longer the definition of atomic mass which has been changed to a standard based on the atomic mass of carbon but to our degree of accuracy, this is good enough). All electrons are accounted for since they simply change locations (thus no need to worry about the difference between atoms and ions). Every other atom in the crystal is sodium and every other atom is chlorine so the average atomic mass of each atom is (23.0 + 35.5)/2 = 29.25 grams per mole. Thus…

$$2.165\frac{g}{cm^3} = \left(2.165\frac{g}{cm^3}\right)\left(\frac{1\ mole}{29.25\ g}\right)\left(\frac{6.022\times10^{23} atoms}{1\ mole}\right) = 4.46\times10^{22} atoms/cm^3$$

    Half of these atoms are sodium, so there are 2.23×10^{22} *sodium atoms per* cm^3.

    b. What is the average distance one sodium ion and a next-door neighbor chlorine ion?

    There are 4.46×10^{22} *atoms per cubic centimeter so each atom occupies an imaginary cubic box of volume of...*

$$Volume = \frac{1\ cm^3}{4.46\times10^{22} atoms} = 2.24\times10^{-23} cm^3$$

    The distance between next-door-neighbor atoms would be given by the length of one side of this imaginary cube:

$$Distance = (2.24\times10^{-23} cm^3)^{\left(\frac{1}{3}\right)} = 2.82\times10^{-8} cm$$

    c. What is the average distance between one sodium ion and the nearest sodium ion?

    The nearest sodium ion would be located diagonally across one face of the cube

$$distance = \sqrt{2}\times(2.82\times10^{-8} cm) = 4.00\times10^{-8} cm$$

7. Great moments in television:

Editor's note: neither of these events are exact quotes from copyrighted television material and are thus not examples of plagiarism! However, both represent paraphrases of actual television moments, great or otherwise.

a. The engineer reported to the captain that the shields were failing. The captain asked the science officer how long they would hold up. The science officer reported that they would last another 47.18 seconds. Without knowing much about shields and photon torpedoes, what do you think of the science officer's answer? Explain your reasoning.

This is just silly. The quote suggests knowledge of the timing of an event in the future to the nearest hundredth of a second when it takes several seconds just to say "forty-seven point one eight seconds."

b. The "world's greatest forensic anthropologist" and the "king of the lab" were at a murder scene. The king of the lab found a worm near the corpse. He said these worms move one foot per day and the worm was 38.3 inches from the body. The world's greatest forensic anthropologist then calculated that this meant the time of death was three days, four hours, and thirty-six minutes ago. Without knowing much about worms and corpses, what do you think of her calculation?

Silly, silly, silly. Does the worm move exactly 1.00 feet per day, accurate to three significant figures? Does it always move with the same speed in a perfectly straight line? Did it leave the body exactly at the time of death? And why would a lab scientist who was not concerned with TV ratings measure the distance in inches, anyway?

8. The speed of sound can be calculated from atmospheric pressure and density. You could look up the formula but let's pretend you don't know how. Standard atmospheric pressure is (very nearly) 100,000 pascals. One pascal is one kilogram per "meter second squared" (1 $kg/(m\,s^2)$). Notice that the seconds are squared but the meters are not. The density of air is about 1.2 grams per liter. One liter is 1000 cm^3.

a. Using *only the information above*, estimate the speed of sound in air.

Speed has dimensions of length per unit time (or units of meters per second). Pressure and density both involve mass (kilograms) which we don't want so we'll have to divide one by the other to get rid of the kilograms.

$$\left[\frac{pressure}{density}\right] = \left[\frac{P}{\rho}\right] = \left[\frac{\left(\frac{M}{L\,T^2}\right)}{\left(\frac{M}{L^3}\right)}\right] = \frac{L^2}{T^2} = \left(\frac{L}{T}\right)^2$$

We want dimensions of L/T so we need "some number" (#) times the square root of this quantity:

$$Speed = (\#) \times \sqrt{\frac{P}{\rho}} = (\#) \times \sqrt{\frac{10^5\ Pa}{1.2\ kg/m^3}} = (\#) \times (290\ m/s)$$

Since we have no knowledge to the contrary our best estimate comes from the assumption that this unknown number is approximately equal to one, and since we are just estimating it only makes sense to report one significant figure:

$$Speed\ of\ sound \approx 300\ m/s$$

b. Now look up a measured value for the speed of sound in air. Calculate the relative error of your estimate. Express your result as a percentage.

Textbooks report that the speed of sound is about 330 m/s, so our estimate was accurate to within about 10%. That's pretty good! We won't always be so lucky with these kinds of estimates but this one works out fairly accurately without too much work.

9. While studying a problem in fluid dynamics, a student came up with some equations involving several variables that have the dimensions listed in the table below. Determine whether each equation (a through d) is dimensionally consistent or inconsistent.
[Note: the symbol r represents a radius and the symbol A represents an area.]

$[m] = M$ $[v] = L\,T^{-1}$	$[t] = T$ $[x] = L$	$[z] = L^2\,T^{-1}$ $[f] = M\,L\,T^{-2}$	$[P] = M\ L^{-1}\,T^{-2}$ $[\eta] = M\ L^{-1}\,T^{-1}$

a) $z = \left(\frac{3}{5}\right)\frac{f}{\eta}$

$[z] = L^2\,T^{-1}$ $\quad\quad$ $\left[\left(\frac{3}{5}\right)\frac{f}{\eta}\right] = (1)\left(\frac{M\ L\,T^{-2}}{M\ L^{-1}\,T^{-1}}\right) = L^2\,T^{-1}$ $\quad\quad$ ***consistent!!!***

b) $f = \sqrt{\pi r^2\,(zP\eta)}$

$[f] = M\ L\,T^{-2}$ $\quad\quad$ $\left[\sqrt{\pi r^2\,(zP\eta)}\right] = \left((1)L^2\left(L^2T^{-1}\ \ ML^{-1}T^{-2}\ \ ML^{-1}T^{-1}\right)\right)^{1/2} = MLT^{-2}$

consistent!

c) $x = \frac{\eta A v}{f} - \frac{mz}{\eta}$ $\quad [x] = L$

$$\left[\frac{\eta A v}{f} - \frac{mz}{\eta}\right] = \frac{ML^{-1}T^{-1}\ L^2\ LT^{-1}}{MLT^{-2}} - \frac{M\ L^2T^{-1}}{ML^{-1}T^{-1}} = L - L^3$$ $\therefore$ **inconsistent!!!**

d) $\frac{f}{4\pi z} - P = Q$ $\quad \left[\frac{f}{4\pi z}\right] = \frac{MLT^{-2}}{L^2T^{-1}} = \boldsymbol{ML^{-1}T^{-1}}$ $\quad [P] = ML^{-1}T^{-2}$

$\therefore$ **inconsistent!!!**

(Notice that we don't need to know what *Q* is. The dimensions on the lefthand side of the equation are inconsistent.)

e) Can a consistent equation be wrong?

$Yes.\ \ An\ example\ would\ be:\ \ Volume\ of\ a\ sphere = \frac{2}{3}\pi r^3$

f) Can an inconsistent equation be right?

No. If the units and dimensions are wrong then the whole thing is wrong.

10. The *Schwarzschild radius* of an object is the radius of a sphere of equivalent mass that would have such intense gravity that light could not escape (the object would be a black hole). The strength of gravity is given by $G = 6.7\times10^{-11}\ m^3\ kg^{-1}\ s^{-2}$. The speed of light is approximately $c = 3.0\times10^{8}\ m/s$. The mass of the Earth is $6.0\times10^{24}\ kg$. Using only this information, estimate the Schwarzschild radius of the Earth. You may compare your answer to the accepted value (which you could find on the Internet) if you wish.

We need an answer with dimensions of length (units of meters) and only length. We don't want kilograms or seconds in our answer. To get rid of kilograms we will have to multiply G by the mass of the earth (m). Then to get rid of "seconds" we will have to divide by the speed of light squared:

$$\left[\frac{G\,m}{c^2}\right] = \frac{\left(\frac{L^3}{M\,T^2}\right)(M)}{\left(\frac{L}{T}\right)^2} = L,\quad so\ our\ estimate\ is\ r \approx \frac{Gm}{c^2} = 0.45\ cm$$

The actual formula which arises from Einstein's equations of general relativity is:

$$r_{Schwartzchild} = \frac{2Gm}{c^2} = 0.9\ cm$$

It turns out that if the mass of the earth really were to be squeezed into such a small volume the density of the result would be greater than the density of a proton or neutron. We know of no way that such a density could be achieved, so the earth does not have enough mass to ever become a black hole.