

Quiz #1: Physics 201

NAME: SOLUTIONS

Do **YOUR OWN WORK** and **SHOW ALL OF IT!**

1) Pirates of the Bathtub

The photo on the left (below) is a picture of the ship Lady Washington (of Aberdeen, Washington) which portrayed the ship “Interceptor” in the film *Pirates of the Caribbean: Curse of the Black Pearl*. The photo on the right is a scale model which also portrayed the Interceptor in scenes shot in a small bathtub with electric fans and scary special effects. For the purposes of this problem we will assume that the model is a perfect replica made of the same materials but $1/N^{\text{th}}$ scale in every direction ($1/N^{\text{th}}$ the length, $1/N^{\text{th}}$ the height, and $1/N^{\text{th}}$ the width).



The real Lady Washington



Model used for some scenes in “Pirates”

- a) The sails on the Lady Washington have a combined area of 4500 square feet. What is the total area of the sails on the model?

Since the sails are measured in units of AREA (dimensions of L^2) the sails on the model will be smaller by a factor of N^2 :

$$\begin{aligned} \text{Area} &= (4500 \text{ sq. ft.}) \left(\frac{1}{N^2} \right) = 5.0 \text{ sq. ft. (if } N = 30) \\ &= 7.2 \text{ sq. ft. (if } N = 25) \end{aligned}$$

- b) Sailors measure the volume of a ship with a measure called “gross tonnage” (don’t worry about it – it is just a measure of volume). The units are the same as the units of weight: there are 2000 pounds in a ton, and so on. The gross tonnage of the bathtub model is 7.0 pounds. What is the gross tonnage of the Lady Washington (in tons where 2000 pounds is exactly one ton)?

Since “gross tonnage” is a measure of VOLUME (dimensions of L^3) the tonnage of the full-sized ship will be larger by a factor of N^3 :

$$\begin{aligned} \text{Tonnage} &= (7.0 \text{ lbs.}) (N^3) \left(\frac{1 \text{ ton}}{2000 \text{ lbs}} \right) = 95 \text{ tons (if } N = 30) \\ &= 55 \text{ tons (if } N = 25) \end{aligned}$$

See next page!

2) Power is required to keep a ship moving for two reasons: (1) to move water out of the way, and (2) to overcome friction between the ship and the water. The power required for a ship to simply overcome friction is related to the viscosity of water, the speed of the ship, and a quantity called the “drag length” of the ship. The “drag length” is not the length of the ship but it has units of length.

(1) The viscosity of seawater is $\eta = 1.00 \times 10^{-3} \frac{\text{kg}}{\text{s m}}$,

(2) The drag length of the Lady Washington is D (about 460 m or 510 m) ,

(3) The top speed of the Lady Washington is about V (either 5.7 m/s or 6.1 m/s) ,

(4) The SI unit of power is the watt: $1 W = 1 \frac{\text{kg m}^2}{\text{s}^3}$

Estimate the power required to overcome friction for the Lady Washington when she is at top speed, and then state whether you think this is a significant fraction of the total power required to keep the Lady Washington moving.

You don't need to know anything about ships or viscosity to answer this question.

We have to combine the quantities in the problem to get units of watts $\left(1 W = 1 \frac{\text{kg m}^2}{\text{s}^3} \right)$.

There is only one way to do this:

$$\text{Power} = (\#) \times \eta D V^2 = (\#) \times (15 \text{ watts})$$

$$\text{or} = (\#) \times (19 \text{ watts})$$

So our estimate would be 10 to 20 watts.

Our estimate might be wrong and the power required to overcome viscosity might be a couple hundred watts, but that's just the power consumed by a couple of bright light bulbs. It doesn't seem possible that a large sailing ship could move with only the power that it takes to light a small room. So we conclude that the power required to overcome viscosity is NOT a significant fraction of the power required to keep the ship moving.

- 3) Checking out equations: Use the information in the table below to determine which of the equations in part a and b are dimensionally consistent.

[frequency] = T^{-1}	[volume] = L^3
[force] = MLT^{-2}	[rotational inertia] = ML^2
[area] = L^2	[torque] = ML^2T^{-2}

For each equation, answer “*consistent*” or “*not consistent*” and show your work.

a) torque = (force) \times $\sqrt{\text{area}}$

$$[\text{torque}] = ML^2T^{-2}$$

$$\left[(\text{force}) \times (\sqrt{\text{area}}) \right] = (MLT^{-2})(L^2)^{1/2} = ML^2T^{-2}$$

So this equation is *consistent*. (Possibly wrong, but consistent.)

b) force \times volume = (rotational inertia) \times (area) \times (frequency)

$$[\text{force} \times \text{volume}] = (MLT^{-2})(L^3) = ML^4T^{-2}$$

$$\left[(\text{rotational inertia}) \times (\text{area}) \times (\text{frequency}) \right] = (ML^2)(L^2)(T^{-1}) = ML^4T^{-1}$$

So this equation is *inconsistent*. (It can't possibly be correct.)