Circuits I: (without inductance):

Resistors (Ohm's Law): $V = IR$ Power: $\text{Power} = VI = I^2 R$

Kirchhoff's Rules:

1. *Junctions:* "$\sum I_{in} = \sum I_{out}$", 2. Loops: $\sum voltages = 0$

Resistors in parallel: $\frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{R_{\text{TOTAL}}}$ …in series: $R_1 + R_2 = R_{\text{TOTAL}}$

Capacitors: $Q = C\,\Delta V$

Capacitors in parallel: $C_1 + C_2 = C_{\text{TOTAL}}$ … in series: $\frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{C_{\text{TOTAL}}}$

RC Circuits: Time constant: $\tau = RC$

Discharging or charging: $f(t) = f_1\, e^{-t/\tau} + f_2$

Often: $f(t) = f_0\, e^{-t/\tau}$ or $f(t) = f_\infty \left(1 - e^{-t/\tau}\right)$

Magnetism: Forces: $\vec{F}_{magnetic} = q\vec{v} \times \vec{B} = \ell \vec{I} \times \vec{B}$ or $\vec{F}_{EM} = q\left[\vec{E} + \left(\vec{v} \times \vec{B}\right)\right]$

Magnetic fields from currents: A distance r from a straight wire: $\left|\vec{B}\right| = \frac{\mu_0}{2\pi}\frac{I}{r}$

At the center of a loop of wire of radius R: $\left|\vec{B}\right| = \frac{\mu_0}{2}\frac{I}{R}$

In a cylindrical coil (or inductor): $\left|\vec{B}\right| = \frac{\mu_0\, N\, I}{length}$

Torque on a loop of wire: $\vec{\tau} = \vec{\mu} \times \vec{B}$ where $\mu = N\, I\, Area$

Inductance: $EMF = -L\frac{dI}{dt}$, and for a cylindrical inductor $\left|\vec{B}\right| = \frac{\mu_0\, N\, I}{length}$

For an LC circuit (capacitor and inductor):

$q = q_{\max} \sin\left(\omega_0 t - \phi\right)$, $I = \dot{q}$, $EMF_L = -L\ddot{q}$, and $\omega_0 = \sqrt{\frac{1}{LC}}$

Maxwell's equations: $\epsilon_0 = 8.854 \times 10^{-12}\ C^2/(N\, m^2)$ $\mu_0 = 4\pi \times 10^{-7}\ T\, m/A$

Gauss' law (for $\vec{E}$ and $\vec{B}$) $\int_{\substack{closed\\surface}} \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon_0}$ **and** $\int_{\substack{closed\\surface}} B \cdot d\vec{A} = 0$

Faraday's law: $\mathcal{E}mf = -\frac{d\Phi_B}{dt}$ or $\int_{loop} \vec{E} \cdot d\vec{\ell} = -\frac{d}{dt}\int \vec{B} \cdot d\vec{A}$

Ampere-Maxwell law: $\int_{loop} \vec{B} \cdot d\vec{\ell} = \mu_0 I_{encloed} + \mu_0\, \epsilon_0\, \frac{d}{dt}\int \vec{E} \cdot d\vec{A}$

Harmonic oscillators:

Angular frequency: $\omega = 2\pi f = \dfrac{2\pi}{T}$

SHO Equation: $\ddot{x} = -\omega^2 x$

Solution to SHO Equation:

$$x - x_{eq} = A\sin(\omega_0 t - \varphi)$$

where ω_0 is the natural *angular* frequency of the oscillator

Mass on a spring: $\omega_0 = \sqrt{\dfrac{k}{m}}$ Simple pendulum: $\omega_0 = \sqrt{\dfrac{g}{\ell}}$

Wave kinematics:

Traveling wave: $f(x,t) = f(x \pm vt)$

Standing wave: $f(x,t) = f(x+vt) \pm f(x-vt)$

Sinusoidal wave: $f(x,t) = A\sin(kx \pm \omega t \pm \varphi)$

$$or\ \ A\sin(kx + \omega t + \varphi) \pm A\sin(kx - \omega t + \varphi)$$

$$k = \frac{2\pi}{\lambda} \quad \omega = \frac{2\pi}{T} \quad v = \frac{\lambda}{T} = \lambda f = \frac{\omega}{k}$$

Wave dynamics:

The wave equation: I forget…

The wave equation is either $\dfrac{\partial^2 f}{dx^2} = v^2\ \dfrac{\partial^2 f}{dt^2}$ or $\dfrac{\partial^2 f}{dt^2} = v^2\ \dfrac{\partial^2 f}{dx^2}$

I can't remember which it is, but you ought to be able to figure it out.

Velocities:

strings: $v = \sqrt{\dfrac{\tau}{\mu}}$ sound: $v = \sqrt{\dfrac{B}{\rho}} = \sqrt{\dfrac{\gamma P}{\rho}}$ light: $v = c = \sqrt{\dfrac{1}{\mu\varepsilon}}$

Sound:

Intensity and sound level (volume):

Intensity $= I = \dfrac{\text{Power}}{\text{Area}}$ $SI\ Unit = \dfrac{watt}{meter^2} = W\ m^{-2}$

Sound level in decibels $= \beta = 10\log_{10}\left(\dfrac{I}{I_0}\right)$ $\left[I_0 = 10^{-12}\ W/m^2\right]$

Doppler Effect for SOUND:

$$f_{observed} = \left(\frac{v_s \pm v_{observer}}{v_s \mp v_{source}}\right) f_{emitted}$$

Damped and/or driven harmonic oscillators:

Damped springs: if $F_{spring} = -kx$ and $F_{friction} = -bv$

Without a driver (motor):

$$x = x_{\max}\, e^{\left(-\frac{bt}{2m}\right)} \sin(\omega t - \phi) \quad \text{where} \quad \omega = \sqrt{\omega_0^2 - \left(\frac{b}{2m}\right)^2}, \; v_{\max} = \omega\, x_{\max}, \quad \text{etc.}$$

With a driver (motor with angular frequency ω):

$$v_{\max} = \frac{(\omega F / m)}{\sqrt{\left(\omega^2 - \omega_0^2\right)^2 + \left(\frac{b\omega}{m}\right)^2}} = \frac{F}{\sqrt{\left(m\omega - \frac{k}{\omega}\right)^2 + (b)^2}}$$

Damped harmonic oscillators and RLC Circuits

Springs with fluid friction: $F_{spring} = -kx$ $F_{friction} = -bv$

Kinematics: $v = \dot{x}$ $a = \dot{v} = \ddot{x}$ and $-kx - bv = ma$

Resonant frequency: $\omega_0 = \sqrt{\frac{k}{m}}$

Damped harmonic oscillation:

$$x = x_{max}\, e^{\left(-\frac{bt}{2m}\right)} sin(\omega t - \varphi)\,, \qquad \omega = \sqrt{\omega_0^2 - \left(\frac{b}{2m}\right)^2}\,, \; v_{max} = \omega\, x_{max}$$

RLC ***series*** circuits: $\Delta V_C = \frac{1}{C} q$, $\Delta V_R = RI$, $\Delta V_L = L\dot{I}$, where $\dot{q} = I$ $\dot{I} = \ddot{q}$

And $\frac{1}{C} q + RI + L\dot{I} = 0$

Resonant frequency: $\omega_0 = \frac{1}{\sqrt{LC}}$

Without a driver (AC source):

$$I = I_{\max}\, e^{\left(-\frac{Rt}{2L}\right)} \sin(\omega t - \phi) \quad \text{where} \quad \omega = \sqrt{\omega_0^2 - \left(\frac{R}{2L}\right)^2}\,, \quad I_{\max} = \omega\, q_{\max}, \quad \text{etc.}$$

Driven damped harmonic oscillators and AC Circuits:

With a driver (AC source with angular frequency ω):

$$EMF = V_{max} \sin(\omega t)$$

$$I_{max} = \frac{V_{max}}{Z} = \frac{V_{max}}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V_{max}}{R\sqrt{1 + Q^2\left(x - \frac{1}{x}\right)^2}}$$

$$= \frac{V_{max}}{R} \cos\varphi$$

Symbols and vocabulary for AC Circuits

Voltage	Reactance	
$\Delta V_L = X_L\, I_{MAX} \cos(\omega t - \varphi)$	$X_L = \omega L$	$\omega = x\, \omega_0$
$\Delta V_R = X_R\, I_{MAX} \sin(\omega t - \varphi)$	$X_R = R$	$Q^2 = \frac{L}{R^2\, C}$
$\Delta V_C = -X_C\, I_{MAX} \cos(\omega t - \varphi)$	$X_C = \frac{1}{\omega C}$	$\tan\varphi = \frac{X_C - X_L}{R}$

Power: $Power_{MAX} = V_{MAX}\ I_{MAX}\ \cos\varphi = \frac{(V_{max})^2}{R}\ \cos^2\varphi$

$$Power_{RMS} = V_{RMS}\ I_{RMS}\ \cos\varphi = \frac{1}{2}\ Power_{MAX}$$

Q Factor: for an RLC circuit, $Q^2 = \frac{L}{R^2\, C}$

Optics:

Index of refraction:

$$v_{light\ in\ material} = \frac{c}{n}\ ,\quad n \geq 1 \qquad \left(c = 3.00 \times 10^8\ m/s = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \right)$$

$$v = \frac{1}{\sqrt{\mu\, \varepsilon}} \approx \frac{1}{\sqrt{\mu_0\, \varepsilon}} = \frac{1}{\sqrt{\mu_0 \left(\kappa\, \varepsilon_0 \right)}} = \frac{c}{\sqrt{\kappa}}\ , \quad \text{so} \quad n \approx \sqrt{\kappa}$$

Reflection: $\theta_{incident} = \theta_{reflected}$ Refraction: $n_1\ \sin\theta_1 = n_2\ \sin\theta_2$

Intensity:

$$Intensity = \frac{Power}{Area} \qquad \text{For waves:} \quad I_{RMS} = \frac{1}{2} I_{\max}$$

$$I_{\max} = \left| \frac{1}{\mu_0} \left(\vec{E}_{\max} \times \vec{B}_{\max} \right) \right| = c\varepsilon_0\ E_{\max}^2 = \frac{c}{\mu_0} B_{\max}^2\ , \qquad E_{\max} = c\ B_{\max}$$

Polarization:

When unpolarized light passes through one polarizer: $I_1 = \frac{1}{2} I_0$

When polarized light passes through a second polarizer rotated at an angle of θ from the first: $I_2 = I_1 \cos^2\theta = \frac{1}{2} I_0\ \cos^2\theta$

Interference of Light (also known as "Physical Optics"):

These formulas assume

1. That the size of the slits ("*a*") and the distance between the slits ("*d*") are both of the same order of magnitude as the wavelength of the light.
2. That the distance from the "source" (whether that is one slit or two slits) to the observation point is many thousands of times greater than *a* or *d*.

Double slit interference (separation of slits = *d*):

Interference pattern = $\cos^2\left(\frac{1}{2} k\, d \sin\theta\right)$

Constructive interference when: $d \sin\theta = n\,\lambda \qquad (n = 0, \pm 1, \pm 2, ...)$

Single slit interference (size of slit(s) = *a*):

Interference pattern = $\left[\frac{\sin\left(\frac{1}{2} k\, a \sin\theta\right)}{\left(\frac{1}{2} k\, a \sin\theta\right)}\right]^2$

Destructive interference when: $a \sin\theta = m\,\lambda \qquad (m = \pm 1, \pm 2, ...)$

Sound and Music

1. Assignment of names to frequencies of sound is always approximate. For example, one symphony conductor might say that "concert A" is 431 Hz and another might say it is 440 Hz, but in practice 425 Hz will still sound like an "A" note.

2. What matters to an orchestra or a band is that everybody has their instruments tuned the same. It is fine if one instrument is tuned to 430 Hz for an A as long as they are all tuned to 430 Hz for an A.

3. Doubling a frequency or dividing it by two moves the note by "one octave" but the name of the note stays the same. If 440 Hz is an A then so is 220 Hz and 880 Hz (as well as 110 Hz, 760 Hz, and so on).

4. Below is an approximate table of musical notes and frequencies. The lower numbers are based on the assumption that 512 Hz (and 256 Hz and so on) is a C. The higher numbers are based on the assumption that 440 Hz (and 220 Hz and so on) is an A.

5. To find a note that is out of the range of the table, multiply or divide by two to get a number that is on the table. For example, a frequency of 1550 Hz will have the same name as a frequency of half that much: 775 Hz which is a G (so 1550 Hz is a G).

Name of note	***Approx. frequency***
A	431 Hz to 440 Hz
A sharp	456 Hz to 466 Hz
B	483 Hz to 494 Hz
C	512 Hz to 523 Hz
C sharp	542 Hz to 554 Hz
D	575 Hz to 587 Hz
D sharp	609 Hz to 622 Hz
E	645 Hz to 659 Hz
F	683 Hz to 698 Hz
F sharp	724 Hz to 740 Hz
G	767 Hz to 784 Hz
G sharp	813 Hz to 831 Hz
A	861 Hz to 880 Hz
A sharp	912 Hz to 932 Hz
B	967 Hz to 988 Hz
C	1024 Hz to 1047 Hz