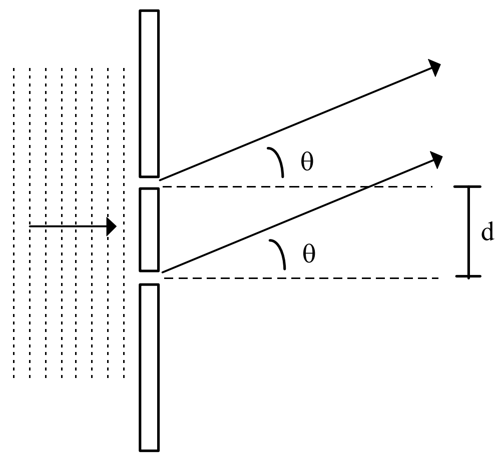
Physics 203
Double Slit Interference

1. Consider the "idealized" double slit interference experiment illustrated schematically below. We imagine that waves from the left (completely in phase) pass through the two very narrow slits and form an interference pattern on a screen that is so far away that the two rays *may be considered to be essentially parallel* even though we are assuming they wind up at a single point on the screen.



- By "idealized" double slit experiment, we assume that the Huygens' waves radiating out from each slit and reaching a point on the distant screen are just circular waves of the form

$$\vec{E} = \vec{E}_0 \cos(kr_{\text{slit}} - \omega t)$$

where r_{slit} is the distance between that point on the screen and that slit.

a) Consider the distance from the point midway between the slits to the point on the screen where the two rays shown meet. Call that distance *r*. Show that the length of the path traveled by the upper light ray is

$$r_{\text{upper}} = r - \tfrac{1}{2}\, d \sin\theta$$

and that the length of the path traveled by the lower light ray is

$$r_{\text{lower}} = r + \tfrac{1}{2}\, d \sin\theta$$

(See next page!)

b) Given the result shown in part a and using the convention that half of the wave comes from the upper slit and half of the wave comes from the lower slit, show that the electric field that reaches a distant point on the screen in the θ direction has the form:

$$\vec{E}_{\text{screen}}(\theta) = \tfrac{1}{2}\vec{E}_0 \cos(kr - \omega t + \tfrac{1}{2} kd \sin\theta) + \tfrac{1}{2}\vec{E}_0 \cos(kr - \omega t - \tfrac{1}{2} kd \sin\theta)$$

c) When dealing with interference "beats" in our discussion of sound, we often used the relationship

$$\cos(a+b) + \cos(a-b) = 2 \cos(a) \cos(b).$$

Use that same relationship here to write the sum of the two electric field waves shown above as the product of an amplitude and two cosine terms.

d) The intensity of a light wave is the amount of power per unit area. A little calculation shows that the ***instantaneous intensity*** of the wave is given by the formula:

$$I(t) = \varepsilon_0 c \left[E(t)\right]^2 \qquad \left[\text{Note: Average Intensity} = I_{AVE} = \frac{1}{2}\varepsilon_0 c\, E_{MAX}^2\right]$$

Show that if there were no interference (that is, if d was zero) the instantaneous intensity of the wave on the screen would be:

$$I_0(t) = \varepsilon_0 c \left(E_0\right)^2 \cos^2(kr - \omega t)$$

e) Now show that in the presence of interference ($d > 0$) the intensity of the wave that reaches the screen in the θ direction is the same as I_0 (and this is true whether you use instantaneous or average intensity) *multiplied by an interference pattern* where the interference pattern is given by

$$\text{Intensity} = I_0 \cos^2\left(\tfrac{1}{2}\, k\, d\, \sin\theta\right)$$

f) Show that this pattern of intensity produces the same result as given in your textbook:

i) bright fringes appear when $d \sin\theta = n\,\lambda, \quad n = 0,1,2,3...$

ii) dark fringes appear when $d \sin\theta = \left(n + \tfrac{1}{2}\right)\lambda, \quad n = 0,1,2,3\ldots$